ME 243
Mechanics of Solids

Lecture 9: Column

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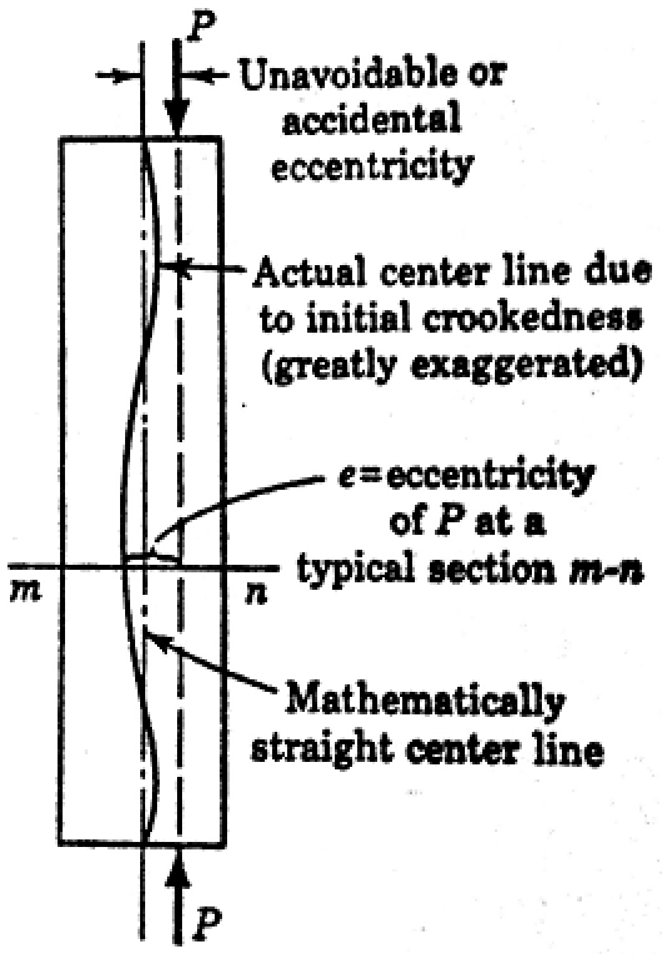
Column

- Column is an object that will absorb compressive axial and eccentric load and its longitudinal dimension is very much greater than lateral dimension (at least 10 times.)
- It is so slender compared to its length that under gradually increasing loads it fails by buckling at loads considerably less than those required to cause failure by crushing.

Column

- An ideal column is assumed to be homogeneous member of constant cross section that is initially straight and is subjected to axial compressive loads.
- Actual columns always have small imperfections of material and fabrication, as well as unavoidable accidental eccentricities of load, which produce the effect shown in fig.

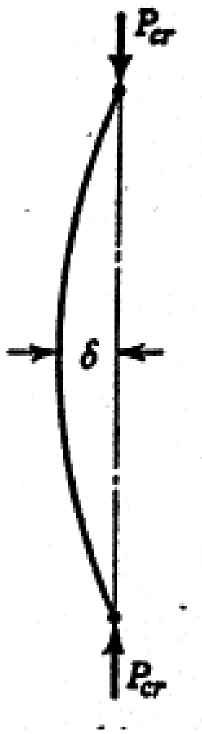


Types of columns

- Long or slender column: long columns fail by buckling or excessive lateral bending.
- Intermediate column: it fails by a combination of buckling and crushing.
- Sometimes the short compression block is also considered as third group. It fails due to crushing.

Critical load

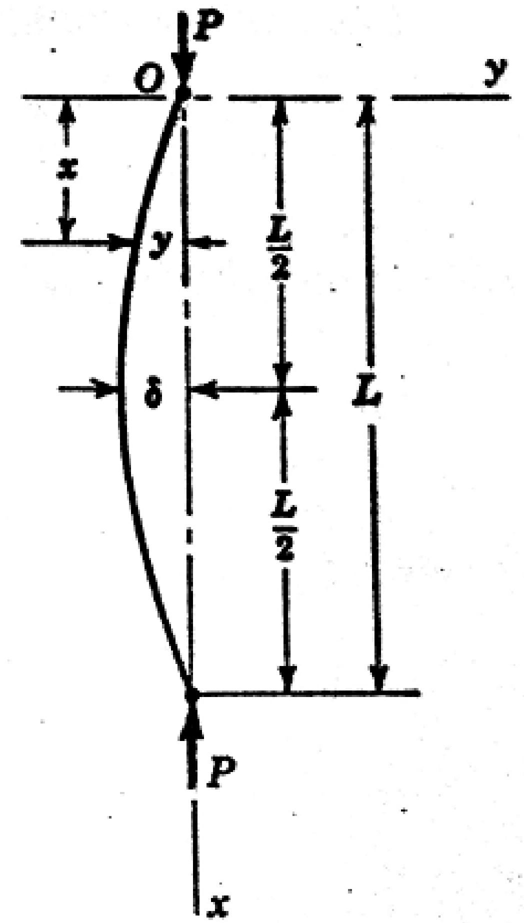
- It is the maximum axial load to which a column can be subjected and still remain straight, although in such an unstable condition that a slight sideways thrust will cause it to bow out.
- Here, the column will elastically fail.



Long column by Euler's formula

- The figure shows the centerline of column in equilibrium under the action of its critical load P. The column has hinged ends restrained against lateral movement.
- The maximum deflection δ is so small that there is no appreciable difference between the original length of the column and its projection on a vertical plane.
- So the slope dy/dx is so small that we may apply the approximate differential equation of the elastic curve of a beam, that is ,

$$EI\frac{d^2y}{dx^2} = M = P(-y) = -Py$$



Long column by Euler's formula

$$EI\frac{d^2y}{dx^2} = M = P(-y) = -Py$$

- This equation can not be integrated directly, because M is not a function of x.
- The equation is similar to the equation of a simple vibrating body,

$$m\frac{d^2x}{dt^2} = -kx$$

- For which, the solution is,

$$x = C_1 \sin\left(t\sqrt{\frac{k}{m}}\right) + C_2 \cos\left(t\sqrt{\frac{k}{m}}\right)$$

Long column by Euler's formula

- By analogy, the solution of our equation can be written as,

$$y = C_1 \sin\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \cos\left(x\sqrt{\frac{P}{EI}}\right)$$

- At x=0, y=0, so we get, $C_2=0$
- At x=L, y=0, so we get,

$$0 = C_1 \sin\left(L\sqrt{\frac{P}{EI}}\right)$$

- Now, either $C_1=0$ (it means no bending in column),

Or, $$L\sqrt{\frac{P}{EI}} = n\pi \qquad (n = 0, 1, 2, 3, \ldots)$$

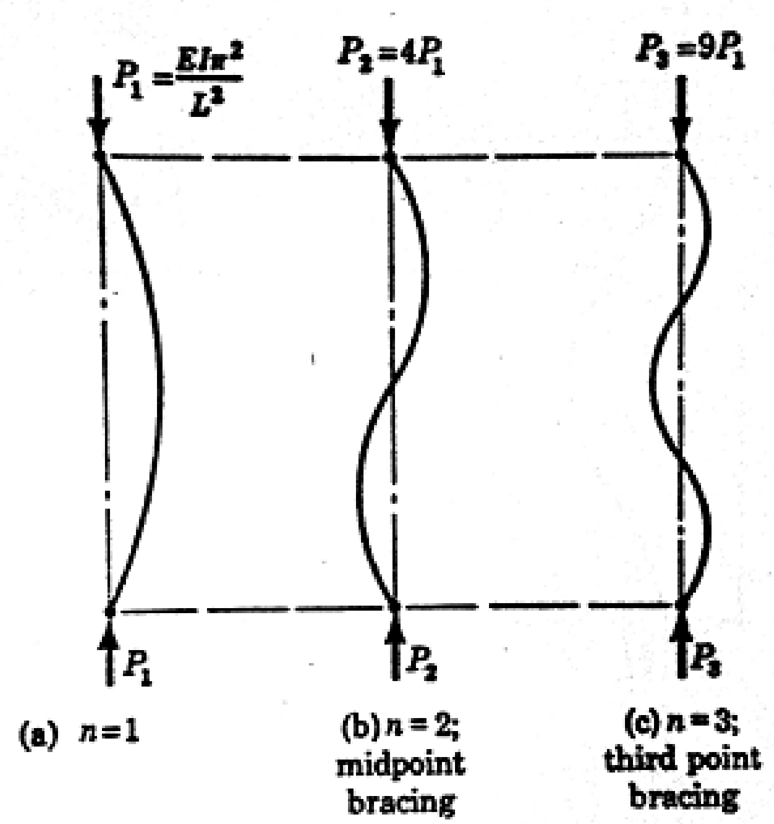
Or, $$P = n^2\frac{EI\pi^2}{L^2}$$

Long column by Euler's formula

$$P = n^2 \frac{EI\pi^2}{L^2}$$

- For n=0, P=0 , which is meaningless.
- For other values of n, the column bends as shown in fig.
- **The most important is fig (a).**
- The others occur with larger loads and are possible only if the column is **braced** at the middle or third points respectively.
- The critical load for a hinged-ended column is therefore,

$$\boxed{P = \frac{EI\pi^2}{L^2}}$$

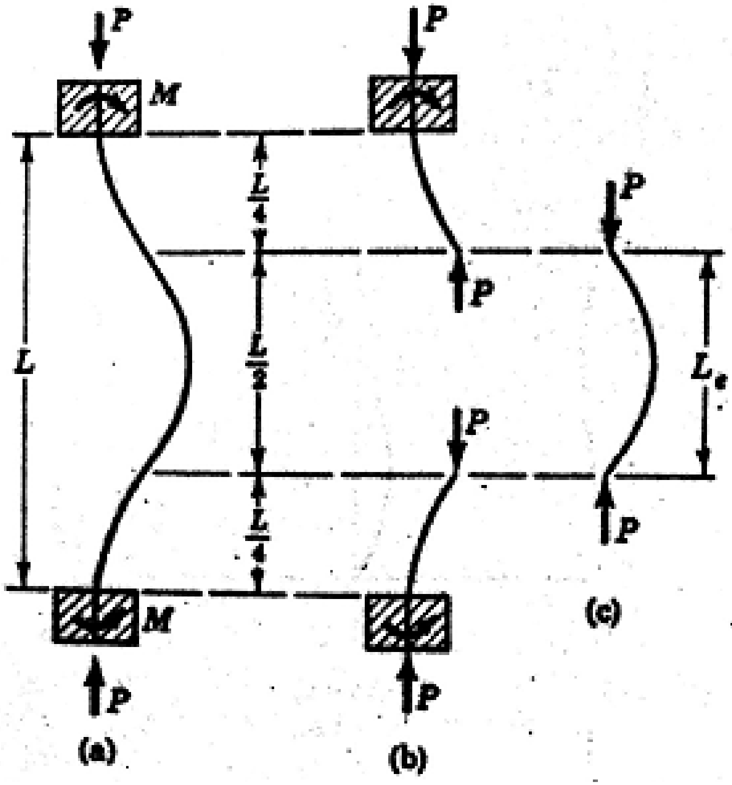


Long column by Euler's formula

- For columns of both ends fixed (fig a), the free body diagram shows that the middle half of the column is equivalent to a hinged column having an effective length $L_e = L/2$, so we get,

$$P = \frac{EI\pi^2}{L_e^2} = \frac{EI\pi^2}{\left(\frac{L}{2}\right)^2} = 4\,\frac{EI\pi^2}{L^2}$$

- This is four times the strength of the column if its ends are hinged.



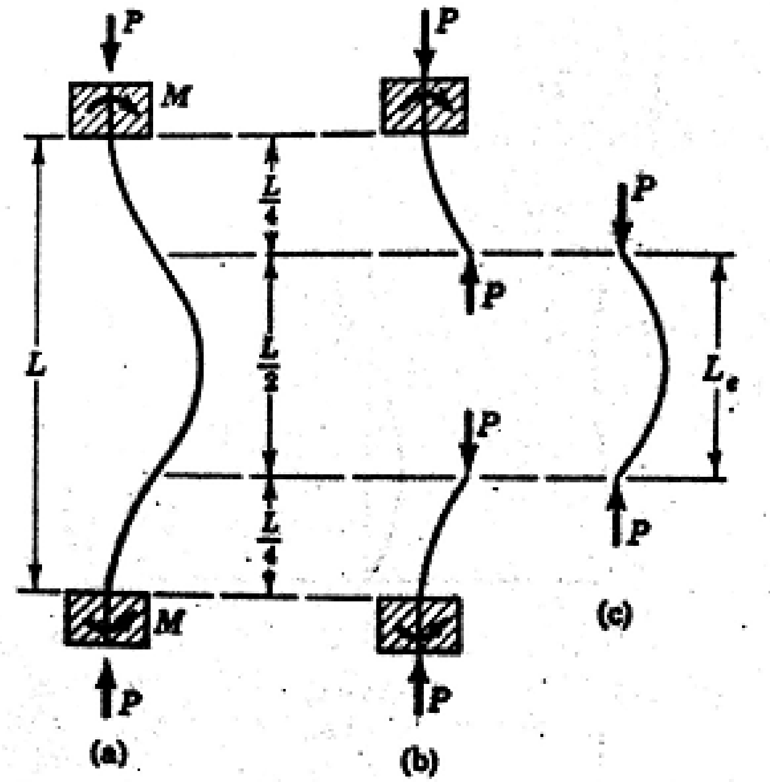
Long column by Euler's formula

- For columns of one end fixed and one end free or flagpole type of column, fig.(b) can be best described. The critical loads on it (fig b) and on the fixed ended column (fig a) are equal, when the fixed ended column is four times as long as the flagpole. So for flagpole column, the effective length,

  $L_e = 0.5L \times 4 = 2L$

So, we get,

$$P = \frac{EI\pi^2}{L_e^2} = \frac{1}{4}\frac{EI\pi^2}{L^2}$$



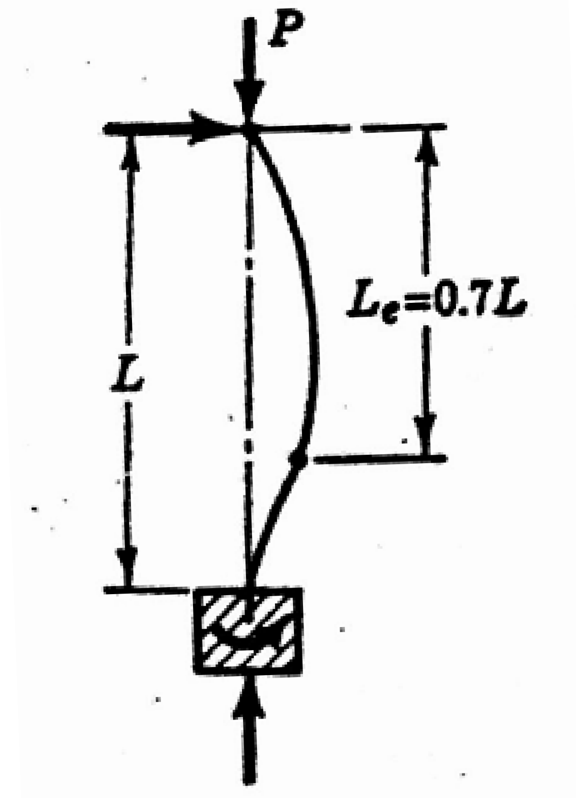
Long column by Euler's formula

- For column with one end fixed and one end hinged, the effective length,

$$L_e = 0.7L$$

So,

$$P = \frac{EI\pi^2}{L_e^2} = \frac{EI\pi^2}{(0.7L)^2} = 2\frac{EI\pi^2}{L^2} \text{ (very nearly)}$$



Long column by Euler's formula

- So in summary , we can write,

$$P = N\frac{EI\pi^2}{L^2} = \frac{EI\pi^2}{L_e^2}$$

End conditions	N= Number of times strength of hinged columns	Le= Effective length
Fixed ends	4	0.5L
One end fixed, the other hinged	2	0.7L
Both end hinged	1	L
One end fixed, the other free	0.25	2L

Euler’s formula: limitations

- The value of ‘I’ in the column formulas is always the least moment of inertia of the cross section. Any tendency to buckle, therefore, occurs about the least axis of inertia of the cross section.
- The critical load that causes buckling depend not on the strength of the material, but only on its dimensions and modulus of elasticity.
- Also for good design, a cross section have as large a moment of inertia as possible.
- The stress accompanying the bending that occurs during buckling must not exceed the proportional limit.

Euler’s formula: limitations

- We can write,

$$I = Ar^2$$

where, I = Least moment of inertia

A= Cross-sectional area

r= Least radius of gyration

For hinged ended columns, Euler’s equation becomes,

$$\frac{P}{A} = \frac{E\pi^2}{\left(\frac{L}{r}\right)^2}$$

Here, (P/A)= average stress in column when carrying critical load, often called critical stress.

(L/r)= slenderness ratio

Euler’s formula: limitations

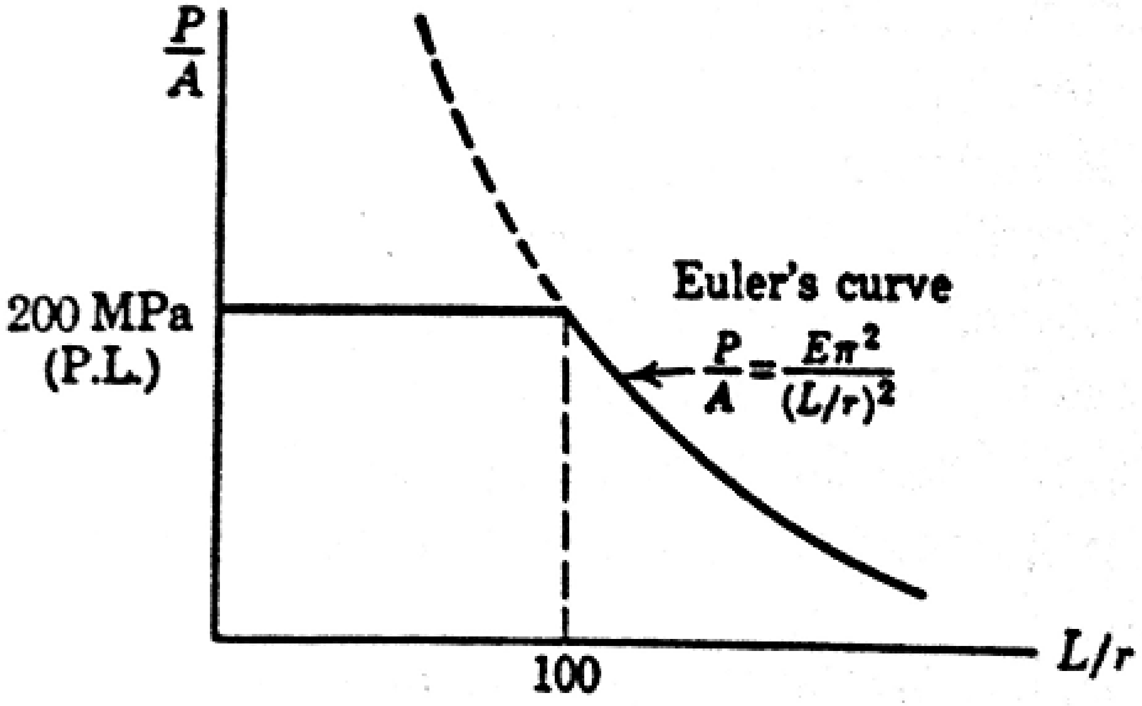
- We define long columns as those for which Euler’s formula can be used. The limiting stress for long column is the proportional limit of the column material.
- Let, for steel, Proportional limit=200 MPa,

  E=200 GPa,

So, (L/r)= 100

- Below the above value of (L/r), the average stress exceeds the proportional limit. So the (L/r)<100, Euler’s formula is not valid.
- The limiting value of slenderness ratio is called critical slenderness ratio.

Euler’s formula: limitations

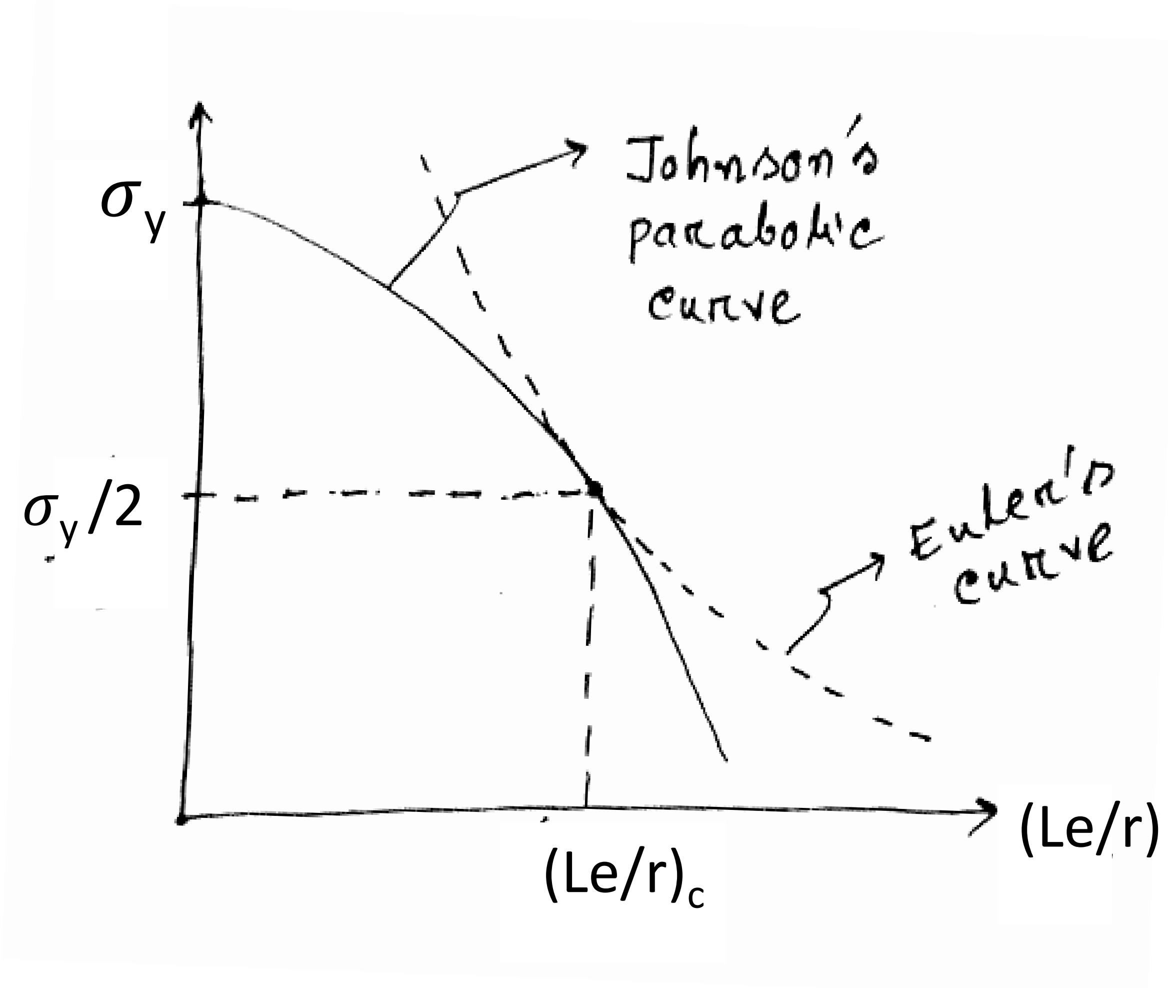


Intermediate column: Johnson's formula

- If the slenderness ratio of a column is less than its critical value, the column is treated as intermediate column or short compression block.
- Several empirical equations are available for intermediate and short columns. Johnson's parabolic formula is mostly used.
- The equation is given by,

$$\frac{P}{A} = \sigma_y - b\left(\frac{Le}{K}\right)^2$$

Johnson's parabolic formula



Johnson's parabolic formula

- The value of (L_e/r) at the intersection of Johnson's curve and Euler's curve is called critical slenderness ratio, $(L_e/r)_c$ or C_o
- At this point, it is found that, $(P/A)= \sigma_y/2$
- So , from Euler's formula,

$$\left(\frac{L_e}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}}$$

- From Johnson's equation, $b = \frac{\sigma_y}{2 C_o^2}$
- So we get, $\boxed{\frac{P}{A} = \sigma_y\left[1 - \frac{(L_e/r)^2}{2 C_o^2}\right]}$

AISC specifications

- American Iron and Steel Construction (AISC) provides the following specifications for columns made of structural steel:
- Critical slenderness ratio:

$$C_0 = \sqrt{\frac{2\pi^2 E}{\sigma_{yp}}}$$

- Euler's equation:

$$\frac{P}{A} = \frac{12\pi^2 E}{23\,(l_e/\pi)^2}$$

- Parabolic equation:

$$\frac{P}{A} = \frac{\sigma_{yp}}{F.S}\left[1 - \frac{(l_e/\pi)^2}{2C_0^2}\right]$$

Where,

$$F.S = \frac{5}{3} + \frac{3}{8}\left(\frac{l_e/\pi}{C_0}\right) - \frac{(l_e/\pi)^3}{8C_0^3}$$

Problem#7.21 (quamrul)

- Select the lightest W shape for a pin ended column of length 4 m that will carry a central load of 450 kN. Use AISC specifications.

$$\sigma_y = 360\ MPa\ \ and\ E = 200\ GPa.$$

(Use table 8.2, page: 561, singer's book)

Problem#E 7.6 (quamrul)

- A column with hinge ends is made of W250×167 section. Find the safe axial load that can be carried when the length is 9m.

$$\sigma_y = 380\ MPa\ \ and\ E = 200\ GPa.$$

- Solve the problem using Euler and Johnson's formula.
- Solve using AISC specifications.

Problem#

- A 6m long column is fabricated from 20 mm thick steel plate to make a square box section of side dimension 100 mm. If the column is fixed at both ends, determine the safe central load that can be carried by the column.

$$\sigma_y = 400\ MPa\ \ and\ E = 200\ GPa.$$